MT4614 Design of Experiments

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19 January 2022

Section 1.3: An example

3 varieties of rye-grass	Cropper
	Melle
	Melba
4 amounts of nitrogen fertilizer	0 kg/ha
	80 kg/ha
	160 kg/ha
	240 kg/ha

2 fields

3 strips of land per field

4 plots per strip

Section 1.3: Layout in the example (2 fields)

0	160	240
160	80	80
80	0	160
240	240	0
↑	↑	↑
Cropper	Melba	Melle

160	80	0
0	160	80
240	0	240
80	240	160
↑	↑	↑
Melba	Cropper	Melle

Varieties are sown on large areas (strips);
fertilizer can be applied to smaller areas (plots).

Section 1.3: Layout in the example (the pattern)

0	160	240
160	80	80
80	0	160
240	240	0
↑	↑	↑
Cropper	Melba	Melle

160	80	0
0	160	80
240	0	240
80	240	160
↑	↑	↑
Melba	Cropper	Melle

The pattern is the combinatorial design:
each variety is on one strip per field;
each quantity of fertilizer is on one plot per strip.

Section 1.3: Layout in the example (lack of pattern)

0	160	240
160	80	80
80	0	160
240	240	0

↑ Cropper ↑ Melba ↑ Melle

160	80	0
0	160	80
240	0	240
80	240	160

↑ Melba ↑ Cropper ↑ Melle

The lack of pattern is caused by randomization:

random order in the allocation of varieties to strips in each field;

random order in the allocation of fertilizer quantities to plots in each strip.

Break for non-technical stuff

All information about MT4614, including timetable, summary of material covered, problem sheets (when available), data files (when needed), is on the web page
`http://www-groups.mcs.st-and.ac.uk/~rab/MT4614/`
There is a direct link to this from MMS.

I encourage you all to write out the notes of each lecture in your own handwriting, with your own comments and explanations added. This is really the only way that the material will get into your brain.

If you have any questions, do not hesitate to email me at
`rab24@st-andrews.ac.uk`
(but do not expect replies during weekends).

Any questions on this lecture so far?

Section 1.4: Defining terms

Definition

An experimental unit is the smallest unit to which a treatment can be applied.

Definition

A treatment is the entire description of what can be applied to an experimental unit.

A pure mathematician would object that these definitions are circular, because they depend on each other, but I have never found a real experiment where we cannot identify treatments and experimental units.

Definition

An observational unit is the smallest unit on which a response will be measured.

Usually the observational units are the same as the experimental units. Sometimes each experimental unit consists of several observational units.

Section 1.4: Defining terms (examples)

Example

In the rye-grass experiment,

$$\text{experimental unit} = \text{observational unit} = \text{plot};$$

$$\text{treatment} = \text{combination of variety with quantity of fertilizer},$$

so the number of treatments is $3 \times 4 = 12$.

Example

Another experiment tests drugs on several patients,
with each patient changing their drug each month.

$$\text{experimental unit} = \text{patient–month combination}.$$

Section 1.4: Calf-feeding experiment

Calves were housed in pens, with ten calves per pen.

Feed D Pen 1 10 calves	Feed C Pen 2 10 calves	Feed D Pen 3 10 calves	Feed B Pen 4 10 calves
Feed B Pen 5 10 calves	Feed A Pen 6 10 calves	Feed A Pen 7 10 calves	Feed C Pen 8 10 calves

Each pen was allocated to a certain type of feed. Batches of this type of feed were put into the pen; calves were free to eat as much of this as they liked. Calves were weighed individually.

treatment = type of feed experimental unit = pen

observational unit = calf

Section 1.4: One more example

Example

In a hand-washing experiment, the treatments are

normal soap and water
water but no soap
hand sanitizer
no washing at all.

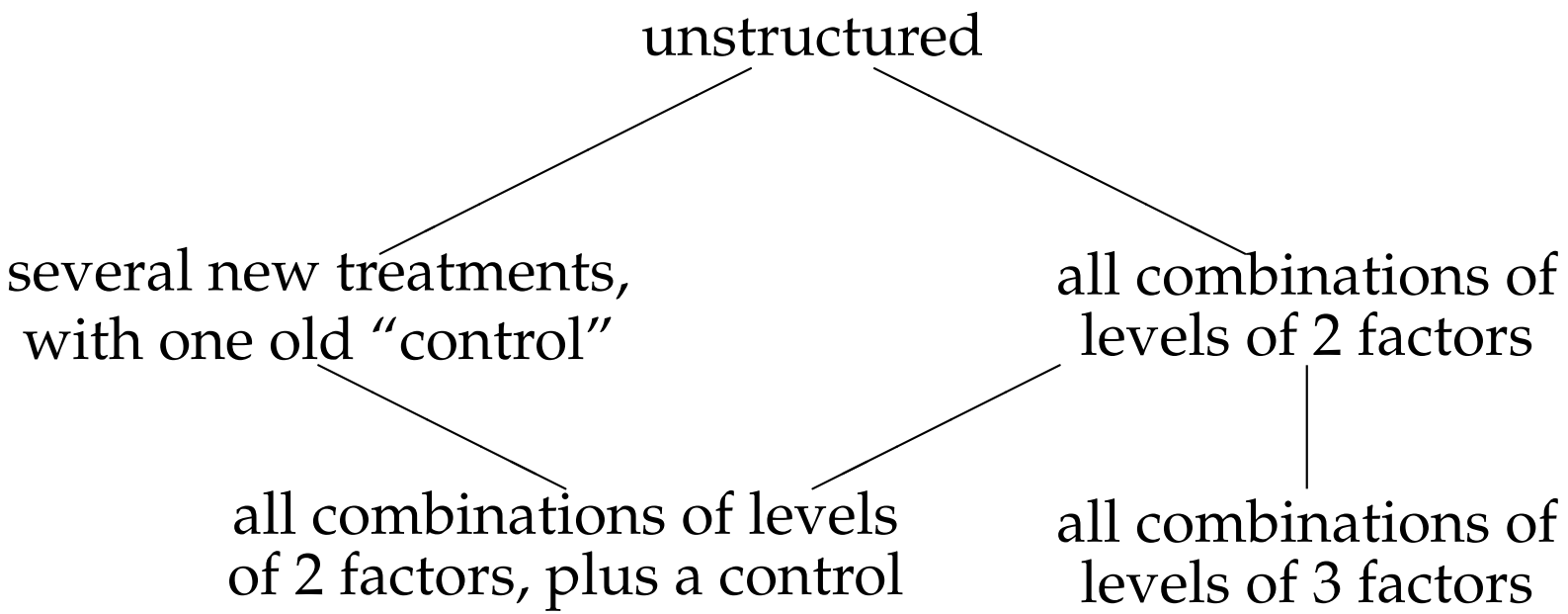
So there are 4 treatments.
(Some people would say that
there are 3 treatments and a control.)

Section 1.4: Notation

notation	meaning
Ω	set of observational units
N	$\lvert\Omega\rvert =$ number of observational units
$\alpha, \beta, \ldots, \omega$	individual observational units
$\mathcal{T}$	set of treatments
$i, j, k, \ldots$	individual treatments
t	$\lvert\mathcal{T}\rvert =$ number of treatments.

Section 1.4: Treatment structure

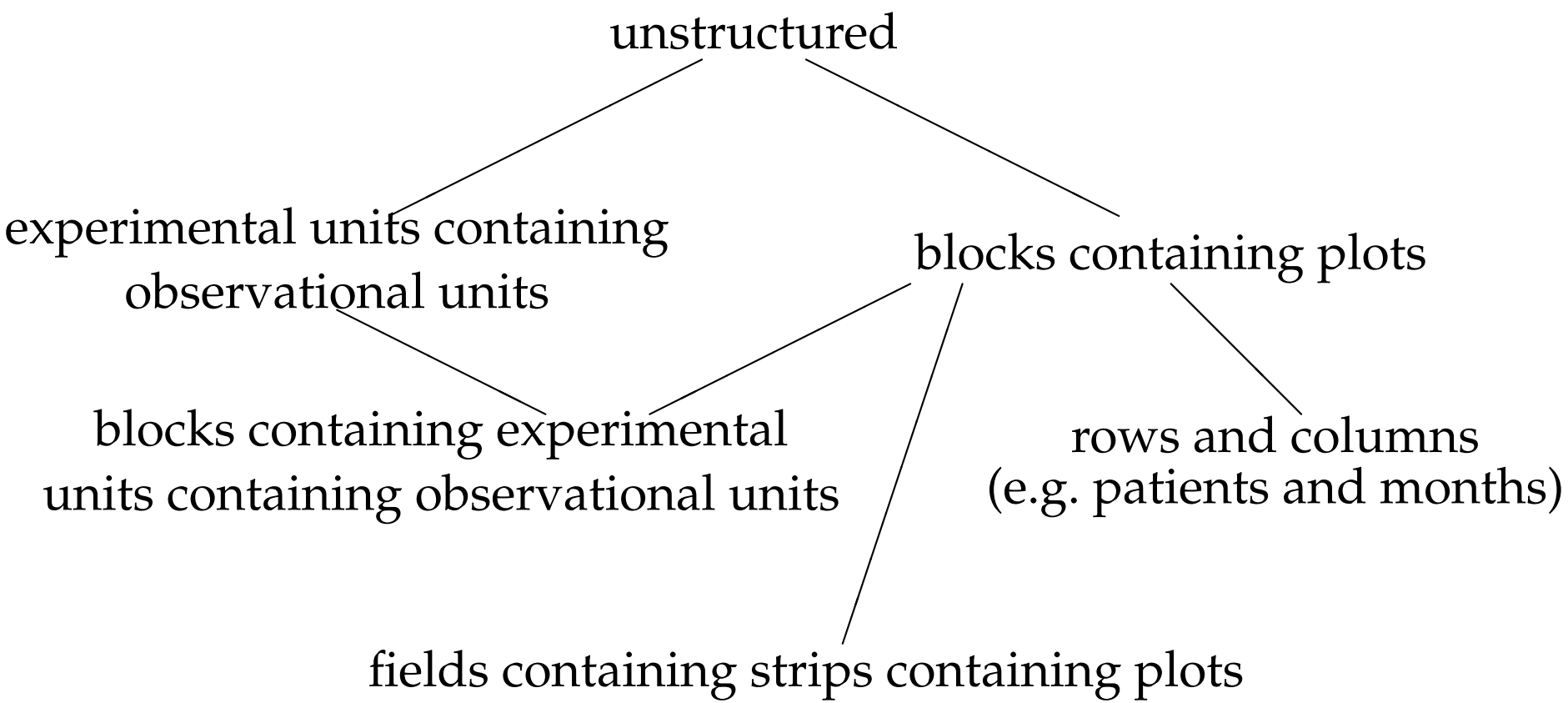
Treatment structure means meaningful ways of dividing up $\mathcal{T}$, such as



. . . and so on.

Section 1.4: Plot structure

Plot structure is meaningful ways of dividing up Ω, such as



Any type of treatment structure can occur with any type of plot structure.

Section 1.4: What is a design?

The design is a function $T\colon \Omega \to \mathcal{T}$.

Observational unit ω gets treatment $T(\omega)$.

Initially, the design has theoretical observational units and coded treatments.

The plan or layout is the design translated into actual observational units and actual treatments, usually by randomization.