SCOTTISH COMBINATORICS MEETING
St Andrews 24–25 April 2017
Abstracts of Talks

Speaker: Laszlo Babai (University of Chicago)

Title: Canonical structures in coherent configurations

Abstract: Coherent configurations (CCs) – non-commutative extensions of association schemes – are a general class of highly regular colorings of the directed complete graph. CCs were first introduced by Schur in 1933 in the context of permutation groups and were later reinvented multiple times in combinatorial, algebraic, and algorithmic contexts.

Large canonical structures, found in all coherent configurations, play a central role as the combinatorial Divide-and-Conquer tools in recent progress on the Graph Isomorphism problem.

Speaker: Rosemary Bailey (St Andrews)

Title: Circular designs balanced for neighbours at distances one and two

Abstract: We consider experiments where the experimental units are arranged in a circle or in a single line in space or time. If neighbouring treatments may affect the response on an experimental unit, then we need a model which includes the effects of direct treatments, left neighbours and right neighbours. It is desirable that each ordered pair of treatments occurs just once as neighbours and just once with a single unit in between. A circular design with this property is equivalent to a special type of quasigroup.

In one variant of this, self-neighbours are forbidden. In a further variant, it is assumed that the left-neighbour effect is the same as the right-neighbour effect, so all that is needed is that each unordered pair of treatments occurs just once as neighbours and just once with a single unit in between.

I shall report progress on finding methods of constructing the three types of design.

Speaker: David Bevan (Strathclyde)

Title: Prolific permutations and permuted packings

Abstract: A permutation of length n is *k-prolific* if each of the $(n-k)$-subsets of the entries in its one-line notation forms a distinct subpermutation. We give a complete characterization of k-prolific permutations for each k, and present an outline of the proof that k-prolific permutations of length n exist for every

$n \geqslant k^2/2 + 2k + 1$, and that none exist of smaller size. Key to these results is a natural bijection between k-prolific permutations and certain *permuted packings* of diamonds.

This talk is based on joint work with Cheyne Homberger and Bridget Tenner.

Speaker: Simon Blackburn (Royal Holloway)

Title: Random Network Coding

Abstract: There has been lots of interest in random network coding from mathematicians and engineers in the past few years, with much of the interest sparked by a paper of Koetter and Kschischang from 2008. The area is an attractive blend of techniques from coding theory, linear algebra and combinatorics, as well as engineering. Ill provide a brief introduction to the area, and Ill highlight some aspects Ive been involved with.

Joint work with Jessica Claridge, and Tuvi Etzion

Speaker: Robert Brignall (Open University)

Title: Staircases, dominoes, and the growth rate of Av(1324)

Abstract: The class of 1324-avoiding permutations is famously hard to count. Whereas every other class that avoids a single length 4 permutation was enumerated in the 1990s, not even the first-order asymptotics (the "growth rate") of Av(1324) is yet known, though Conway and Guttmann suggest that the growth rate is 11.60 ± 0.01 (i.e. there are approximately 11.60^n 1324-avoiders of length n), based on numerical evidence. Rigorous bounds on the growth rate, however, remain somewhat far apart: the best lower bound of 9.81 is due to Bevan, and the best upper bound is 13.74, due to Bona.

As has been known for some time (implicitly, at least), the structure of 1324-avoiding permutations can be loosely characterised in terms of a descending 'staircase' of cells, alternating between the classes Av(132) and Av(213), both of which are well-understood. The shortest non-trivial staircase, comprising just two cells, is called a 'domino', and I will show how to establish that this subclass of Av(1324) has growth rate equal to $27/4 = 6.75$.

By 'gluing' dominoes together to form staircases, this result almost instantly gives a new upper bound on the growth rate of Av(1324) of 13.5, and with rather more work a new lower bound of 10.125. If time permits, I will also describe some refinements to the typical structure of a large domino, giving an improved lower bound for Av(1324) of 10.24.

This is joint work with David Bevan (Strathclyde), Andrew Elvey Price (Melbourne) and Jay Pantone (Dartmouth College).

Speaker: Anders Cleasson

Title: Counting with sign-reversing involutions

Abstract: Combinatorial proofs and negative terms in sums they are applied to don't generally go well together. I will give examples from my own research, as well as other's, of how one can deal with this situation using so-called sign-reversing involutions, which cancel most of the terms. These examples include the counting of ballots, certain posets, and patterns in permutations. I'll also use a sign-reversing involution to explain what the multiplicative inverse of a combinatorial species is.

Speaker: Max Gadouleau (Durham)

Title: An introduction to Finite Dynamical Systems

Abstract: We are interested in complex networks of interacting entities (such as genes, neurons, persons, computers, etc.), where each entity has a finitely valued state and a function which updates the value of the state. Since entities influence each other, this local update function depends on the states of some of the entities. Such a network is called a Finite Dynamical System (FDS). The main problem when studying an FDS is to determine its dynamics given a limited knowledge of it; for instance, we may only know the interaction graph, i.e. which entities influence each other. In this talk, we will review some of the seminal results in the theory of FDSs and focus on the maximisation of images for a given interaction graph.

Speaker: Kitty Meeks (Glasgow)

Title: The interactive sum choice number of graphs

Abstract: List colouring is a natural extension of the basic graph colouring problem, in which each vertex v is given a list L_v of permitted colours and we want to find a proper colouring in which every vertex receives a colour from its list. The choice number of a graph G is the smallest k such that, if every vertex has a list of k permitted colours (chosen by an adversary) we are sure to be able to find a proper list colouring. However, a graph G may have large choice number due to a small part of G (e.g. a clique) that is "hard" to colour, even if the rest of the graph is "easy" to colour. The sum choice number of a graph G (introduced by Isaac in 2002) instead captures the "average difficulty" of colouring the graph, as each vertex v is allowed to have a different list length $f(v)$: we still require

that, however our adversary assigns colours to the lists, there will always be a proper list colouring of G, but our goal is now to minimise the sum of the list lengths.

We introduce a variant of the sum choice number, called the interactive sum choice number. Instead of deciding in advance what the list length should be for each vertex, we can now request colours to be added to the vertices' colour-lists one at a time, so that we are able to make use of the information about the colours assigned so far to determine our future choices; the interactive sum choice number is the total number of colours (summed over all vertices) that we have to request, in the worst case, in order to obtain a proper list colouring. It is clear that the interactive sum choice number cannot exceed the sum choice number; we conjecture that, except in the case of complete graphs, the interactive sum choice is always strictly smaller than the sum choice number.

This conjecture is still wide open, but in this talk I will provide some supporting evidence in its favour: we can show that the conjecture holds in a number of special cases, and indeed that in many of these cases the difference between the two quantities grows as a linear function of the number of vertices.

This is joint work with Marthe Bonamy (Bordeaux).

Maura Paterson (Birkbeck, University of London)

Title: Recent results on strong external difference families

Abstract: An $(n, m, k; \lambda)$-strong external difference family (SEDF) is a set of m disjoint k-subsets $A_1, ...A_m$ of an additive group G of order n with the property that for every subset A_i (with i between 1 and m) we have that each nonzero element of G occurs precisely λ times as a difference of the form $a_i - a_j$ with $a_i \in A_i$ and $a_j \in A_j$ for some $j \neq i$. This definition was recently proposed in connection with the study of optimal algebraic manipulation detection codes, which have applications in cryptography and coding theory. During the past year SEDFs have received an increasing amount of attention, and a range of bounds, constructions and non-existence results have been obtained. In this talk we survey recent progress in this area and highlight some open questions.

Speaker: Mehdi Aghabali (Edinburgh)

Title: Assignment of an appropriate direction to the edges of a given graph to obtain a digraph with maximum 2-factors

Abstract: We give an upper bound on the number of perfect matchings in simple graphs with a given number of vertices and edges. We apply this result to give an upper bound on the number of 2-factors in a directed complete bipartite balanced

graph on $2n$ vertices. The upper bound is sharp for even n. For odd n we state a conjecture on a sharp upper bound.

Speaker: Robert Hancock (Birmingham)

Title: Independent sets in hypergraphs and Ramsey properties of graphs and the integers

Abstract: Many important problems in combinatorics and other related areas can be phrased in the language of independent sets in hypergraphs. Recently Balogh, Morris and Samotij, and independently Saxton and Thomason developed very general container theorems for independent sets in hypergraphs; both of which have seen numerous applications to a wide range of problems. We use the container method to prove results that correspond to problems concerning tuples of disjoint independent sets in hypergraphs.

We generalise the random Ramsey theorem of Rödl and Ruciński by providing a resilience analogue. This result also implies the random version of Turán's theorem due to Conlon and Gowers, and Schacht. We prove a general subcase of the asymmetric random Ramsey conjecture of Kohayakawa and Kreuter. Both of these results in fact hold for uniform hypergraphs. We also strengthen the random Rado theorem of Friedgut, Rödl and Schacht by proving a resilience version of the result.

Speaker: Matt McDevitt (St Andrews)

Title: Insertion Relations on Words

Abstract: Using transducers, we introduce a class of relations on words which generalise the subword order, and discuss the decidability of their properties.

Speaker: Hattie Serocold (Lancaster)

Title: Coordinated isostatic frameworks

Abstract: A bar-joint framework (G, p) is a graph G with an embedding of its vertices into R^d. A framework is said to be isostatic if it is both infinitesimally rigid and the bars are independent. Coordinated frameworks are those with a subset of bars that may extend or retract, but must all do so at once, unlike standard frameworks in which the bar lengths are fixed by the embedding p. We shall look at an inductive construction of isostatic coordinated frameworks, and characterise isostatic frameworks in 2 dimensions with 1 set of coordinated bars.