1. Show that the following two subspaces of $\mathbb{C}^{1 \times 3}$ are equal:
   - $\text{Span}([1, 0, -1], [0, 2, 1], [1, 2, 0])$
   - $\{ [x, y, z] \in \mathbb{C}^{1 \times 3} \mid z = y/2 - x \}$.
   Determine their dimension.

2. Show that $\mathbb{C}^{1 \times 3}$ has the following direct sum decomposition:
   \[ \mathbb{C}^{1 \times 3} = \text{Span}([1, 2, 3]) \oplus \{ [x, y, z] \in \mathbb{C}^{1 \times 3} \mid z = x - y \}. \]

3. Let $V := \mathbb{C}^{1 \times 3}$ and $W := \text{Span}([1, 1, 1])$. Find a complement of $W$ in $V$, that is, a subspace $U$ of $V$ such that $V = W \oplus U$.

4. Let $L := \mathbb{R}^{1 \times 3}$ be the 3-dimensional real row space with the following product:
   \[ ([a, b, c], [x, y, z]) = [a, b, c] \times [x, y, z] := [bz - cy, cx - az, ay - bx]. \]
   Show that this product fulfills the Jacobi identity.

5. Let $L$ be a Lie algebra over a field $\mathbb{F}$ and $H$ a subspace of $L$ (not necessarily a subalgebra). Use the Jacobi identity to show that both the normaliser $N_L(H)$ and the centraliser $C_L(H)$ are Lie subalgebras of $L$.

6. Let $L = \mathbb{C}^{n \times n}$ with $n \geq 2$. Show that the subspace $K$ of skew-symmetric matrices, i.e. $\{ A \in \mathbb{C}^{n \times n} \mid A^t = -A \}$ where $A^t$ is the transposed matrix of $A$, is not an ideal.

7. Let $L$ be any Lie algebra over a field $\mathbb{F}$. Show that the members of the lower central series
   \[ L = L^0 \supseteq L^1 = [L, L] \supseteq L^2 \supseteq \cdots \]
   are in fact ideals in $L$.

8. Let $L$ be any Lie algebra over a field $\mathbb{F}$. Show that the members of the derived series
   \[ L = L^{(0)} \supseteq L^{(1)} = [L, L] \supseteq L^{(2)} \supseteq \cdots \]
   are in fact ideals in $L$. 