1. Fill in the details of the proof of Theorem 8.2(i) to show that $E^{(n)}$ is cyclic.

2. In the following, assume we are working in a field over which the cyclotomic polynomial $Q_n$ is defined. Find $Q_n(x)$, in its simplest form, for
   (i) $n = 8$;
   (ii) $n = 20$.

3. Express $F_8$ using
   - root adjunction;
   - the fact that it is the 7th cyclotomic field over $F_2$.
   Draw up a table showing how the two representations correspond.

4. In this question, you are given the following theorem:
   As in Theorem 7.16, let $I(q, n; x)$ be the product of all monic irreducible polynomials in $F_q[x]$ of degree $n$. Then for $n > 1$ we have
   $$I(q, n; x) = \prod_m Q_m(x),$$
   where the product is extended over all positive divisors $m$ of $q^n - 1$ for which $n$ is the multiplicative order of $q$ modulo $m$, and where $Q_m(x)$ is the $m$th cyclotomic polynomial over $F_q$.
   (i) Using the given theorem (or otherwise), calculate $I(3, 2; x)$.
   (ii) Use part (i) to determine all three monic irreducible polynomials in $F_3[x]$ of degree 2. (Hint: Theorems 8.8 and 8.12 of the notes will help you here).