Is there a Sudoku puzzle with 16 hints?

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Can we choose 16 to make a puzzle?
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Is there a Sudoku puzzle with 16 hints?
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A possible attack

\[
\binom{81}{16} = 33,594,090,947,249,085 \text{ (quadrillions)}
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\[ \implies \text{do not even think about trying all!} \]
A possible attack

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\[ \Rightarrow \text{do not even think about trying all!} \]

Idea: We do not have to try all choices.
A possible attack

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\[ \Rightarrow \text{ do not even think about trying all!} \]

Idea: We do not have to try all choices.

We need constraints that the selection of 16 has to fulfill.
### Unavoidable sets

Any set of 16 hints cannot avoid all of the yellow positions. Because this Sudoku problem has more than one solution.

<table>
<thead>
<tr>
<th>7 9 3</th>
<th>6 8 4</th>
<th>5 1 2</th>
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<tbody>
<tr>
<td>4 8 6</td>
<td>5 1 2</td>
<td>9 3 7</td>
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<td>1 2 5</td>
<td>9 7 3</td>
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Unavoidable sets

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The Plan

- Find lots of **unavoidable sets**.
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- For each solution run a **Sudoku solver** and find another as the known solution.
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- Repeat the same for all other 5,472,730,538 (billions) essentially different filled Sudoku grids.
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Problems:
- Have a program to find 513 unavoidable sets in < 0.1s.
The Plan

- Find lots of unnecessary sets.
- Solve the constraint satisfaction problem to find all subsets of 16 positions intersecting all unnecessary sets.
- For each solution run a Sudoku solver and find another as the known solution.
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Problems:

- Have a program to find 513 unnecessary sets in < 0.1s.
- Find all 767 solutions for 16-subsets in 21min.
The Plan

- Find lots of unavoidable sets.
- Solve the constraint satisfaction problem to find all subsets of 16 positions intersecting all unavoidable sets.
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Problems:

- Have a program to find 513 unavoidable sets in < 0.1s.
- Find all 767 solutions for 16-subsets in 21min.
- Have a Sudoku solver which solves a Sudoku in \( \approx 28 \mu s \) or \( \approx 45000 \) clock cycles.
The Plan

- Find lots of **unavoidable sets**.
- Solve the **constraint satisfaction problem** to find all subsets of 16 positions intersecting all unavoidable sets.
- For each solution run a **Sudoku solver** and find another as the known solution.
- Repeat the same for all other 5,472,730,538 (billions) essentially different filled Sudoku grids.

Problems:

- Have a program to find **513 unavoidable sets** in \(< 0.1s\).
- Find all **767 solutions** for 16-subsets in **21min**.
- Have a **Sudoku solver** which solves a Sudoku in \(\approx 28\mu s\) or \(\approx 45000\) clock cycles.
- This needs an estimated amount of 
  \[6.9 \cdot 10^{12}\] CPU seconds (218659 million years)!