Is there a Sudoku puzzle with 16 clues?

Max Neunhöffer

University of St Andrews

Aberdeen 24.3.2010
Is there a Sudoku puzzle with 16 clues?

Max Neunhöffer

The Problem

What is a Sudoku?

How many clues?

Symmetry

Equivalent Sudokus

Symmetry Breaking

Backtrack Search

Unavoidable Sets

The Hitting Set Problem

Backtrack Search

Random Search

Finding Unavoidable Sets

The Result

Sudoku Grids

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Is there a Sudoku puzzle with 16 clues?

Max Neunhöffer

The Problem
What is a Sudoku?

Rule
Each row, column and 3 × 3-block contains the numbers 1 to 9 each exactly once.
**Is there a Sudoku puzzle with 16 clues?**

Max Neunhöffer

**The Problem**

What is a Sudoku?

How many clues?

**Symmetry**

Equivalent Sudokus

Symmetry Breaking

**Backtrack Search**

Unavoidable Sets

The Hitting Set Problem

Backtrack Search

**Random Search**

Finding Unavoidable Sets

**The Result**

---

### Sudoku Puzzles

![Sudoku Puzzle](image)

**Rule**

Each row, column and $3 \times 3$-block contains the numbers 1 to 9 **each exactly once.**
Is there a Sudoku puzzle with 16 clues?

Max Neunhöffer

The Problem

What is a Sudoku?
How many clues?

Symmetry
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Sudoku Puzzles

Rule
Each row, column and 3 × 3-block contains the numbers 1 to 9 each exactly once.
It is guaranteed that there is a unique solution.
Sudoku Puzzles

Each row, column and $3 \times 3$-block contains the numbers 1 to 9 each exactly once. It is guaranteed that there is a unique solution.
### Solving Sudokus

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- **Is there a Sudoku puzzle with 16 clues?**
- **Max Neunhöffer**

### The Problem

**What is a Sudoku?**

**How many clues?**

### Symmetry

**Equivalent Sudokus**

**Symmetry Breaking**

### Backtrack Search

**Unavoidable Sets**

**The Hitting Set Problem**

**Backtrack Search**

### Random Search

**Finding Unavoidable Sets**

### The Result

- **How difficult is a Sudoku puzzle?**
  - Depends on how much one has to try.
  - A computer solves this in $\approx 28 \mu s \approx 45000$ clock cycles!
Is there a Sudoku puzzle with 16 clues?

Max Neunhöffer

The Problem
What is a Sudoku?
How many clues?

Symmetry
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Solving Sudokus

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The Problem

What is a Sudoku?
How many clues?

Symmetry
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Symmetry Breaking

Backtrack Search
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The Hitting Set Problem
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Random Search
Finding Unavoidable Sets

The Result

Solving Sudokus

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How difficult is a Sudoku puzzle?

−→ Depends on how much one has to try.

A computer solves this in \( \approx 28 \mu s \approx 45000 \) clock cycles!
Is there a Sudoku puzzle with 16 clues?

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The Problem

What is a Sudoku?
How many clues?

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See the table for the complete Sudoku puzzle.

How difficult is a Sudoku puzzle?
- Depends on how much one has to try.
  - A computer solves this in \( \approx 28 \mu s \approx 45000 \text{ clock cycles!} \)
### Solving Sudokus

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Solving Sudokus

How difficult is a Sudoku puzzle?
Is there a Sudoku puzzle with 16 clues?

Max Neunhöffer

The Problem

What is a Sudoku?

How many clues?

Symmetry

Equivalent Sudokus

Symmetry Breaking

Backtrack Search

Unavoidable Sets

The Hitting Set Problem

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Random Search

Finding Unavoidable Sets

The Result

Solving Sudokus

4 2

5 6 4

8 3

1 9

3 4 2

5 1

8 7

How difficult is a Sudoku puzzle?

→ Depends on how much one has to try.
Solving Sudokus

How difficult is a Sudoku puzzle?

→ Depends on how much one has to try.

A computer solves this in $\approx 28 \mu s \approx 45000$ clock cycles!
Is there a Sudoku puzzle with 16 clues?

Max Neunhöffer

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The Result

This puzzle has 17 clues. None of them can be left out.

Question:
Are there 16 clues which uniquely define a Sudoku grid?
Is there a Sudoku puzzle with 16 clues?

Max Neunhöffer

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This puzzle has 17 clues.
Is there a Sudoku puzzle with 16 clues?

Max Neunhöffer

The Problem
What is a Sudoku?
How many clues?

Symmetry
Equivalent Sudokus
Symmetry Breaking

Backtrack Search
Unavoidable Sets
The Hitting Set Problem
Backtrack Search

Random Search
Finding Unavoidable Sets

The Result

This puzzle has 17 clues. None of them can be left out.
Is there a Sudoku puzzle with 16 clues?

This puzzle has 17 clues. None of them can be left out.

Question:
Are there 16 clues which uniquely define a Sudoku grid?
What is known?

- There are altogether
  
  \[6670\,903\,752\,021\,072\,936\,960 \approx 6.671 \cdot 10^{21}\]

  different full Sudoku grids.
What is known?

- There are altogether
  
  \[ 6,670,903,752,021,072,936,960 \approx 6.671 \cdot 10^{21} \]
  
dozen different full Sudoku grids.

- **Gordon Royle** (University of Western Australia, Perth) maintains a collection of currently **49151** pairwise inequivalent Sudoku puzzles with 17 clues. 

What is known?

- There are altogether
  
  \[6 670 903 752 021 072 936 960 \approx 6.671 \cdot 10^{21}\]
  
  different full Sudoku grids.

- **Gordon Royle** (University of Western Australia, Perth) maintains a collection of currently 49151 pairwise inequivalent Sudoku puzzles with 17 clues.
  

- He reckons “that new 17-clue Sudoku puzzles are becoming rarer to find”.
What is known?

- There are altogether
  \[6\,670\,903\,752\,021\,072\,936\,960 \approx 6.671 \cdot 10^{21}\]
  different full Sudoku grids.

- Gordon Royle (University of Western Australia, Perth) maintains a collection of currently 49151 pairwise inequivalent Sudoku puzzles with 17 clues.
  
  [Link](http://www.csse.uwa.edu.au/~gordon/sudokumin.php)

- He reckons “that new 17-clue Sudoku puzzles are becoming rarer to find”.

- There is a set of 16 clues which allows exactly two solutions.
Equivalence of Sudokus

Equivalence transformations:
Is there a Sudoku puzzle with 16 clues?

Max Neunhöffer

The Problem
What is a Sudoku?
How many clues?

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Equivalent Sudokus
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Equivalence of Sudokus

Equivalence transformations:

- **Permute**: rows in a block,
Equivalence of Sudokus

Equivalence transformations:

- **Permute:** rows in a block, columns in a block,
Equivalence of Sudokus

Equivalence transformations:

- **Permute**: rows in a block, columns in a block, block-rows,
Is there a Sudoku puzzle with 16 clues?

Max Neunhöffer

The Problem
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Is there a Sudoku puzzle with 16 clues?

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Finding Unavoidable Sets

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### Equivalence transformations:

- **Permute**: rows in a block, columns in a block, block-rows, block-columns
- **Reumber**: entries

![Sudoku Grid](image)
Equivalence of Sudokus

Equivalence transformations:

- **Permute**: rows in a block, columns in a block, block-rows, block-columns
- **Renumber**: entries
- **Flip**: entire grid
Equivalence of Sudokus

Equivalence transformations:

- **Permute**: rows in a block, columns in a block, block-rows, block-columns
- **Renumber**: entries
- **Flip**: entire grid

→ All concatenations of these form a **group**.
Equivalence of Sudokus

**Definition: Equivalent Sudokus**

Two Sudoku grids/puzzles are called **equivalent** if one arises from the other by applying a sequence of equivalence transformations.
Definition: Equivalent Sudokus

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Equivalence of Sudokus

Definition: Equivalent Sudokus

Two Sudoku grids/puzzles are called equivalent if one arises from the other by applying a sequence of equivalence transformations.

We form equivalence classes or orbits.
The Problem
What is a Sudoku?  How many clues?

Symmetry
Equivalent Sudokus  Symmetry Breaking

Backtrack Search
Unavoidable Sets  The Hitting Set Problem  Backtrack Search

Random Search
Finding Unavoidable Sets

The Result

Equivalence of Sudokus

Definition: Equivalent Sudokus

Two Sudoku grids/puzzles are called equivalent if one arises from the other by applying a sequence of equivalence transformations.

We form equivalence classes or orbits.

→ There are 5472730538 classes (Russell/Jarvis 2006)

http://www.afjarvis.staff.shef.ac.uk/sudoku/
Symmetry Breaking

We “break the symmetry” by considering exactly one from each equivalence class.
Symmetry Breaking

We “break the symmetry” by considering exactly one from each equivalence class. Consider only first block row:

1. We can renumber to get this left hand 3 × 3-block:

```
1 2 3 | 4 6 8 | 9 5 7
4 5 6 | 9 1 7 | 8 3 2
7 8 9 | 3 5 2 | 1 4 6
```
Symmetry Breaking

We “break the symmetry” by considering exactly one from each equivalence class. Consider only first block row:

1. We can renumber to get this left hand $3 \times 3$-block:

\[
\begin{array}{ccc}
1 & 2 & 3 \\
4 & 5 & 6 \\
7 & 8 & 9
\end{array}
\begin{array}{ccc}
4 & 6 & 8 \\
9 & 1 & 7 \\
3 & 5 & 2
\end{array}
\begin{array}{ccc}
9 & 5 & 7 \\
8 & 3 & 2 \\
1 & 4 & 6
\end{array}
\]

2. Distinguish cases for first row:

(Type I)

\[
\begin{array}{ccc}
1 & 2 & 3 \\
4 & 5 & 6 \\
7 & 8 & 9
\end{array}
\begin{array}{c}
\{4,5,6\} \\
\{7,8,9\}
\end{array}
\]
Symmetry Breaking

We “break the symmetry” by considering exactly one from each equivalence class. Consider only first block row:

1. We can renumber to get this left hand 3 × 3-block:

\[
\begin{array}{ccc|ccc|ccc}
1 & 2 & 3 & 4 & 6 & 8 & 9 & 5 & 7 \\
4 & 5 & 6 & 9 & 1 & 7 & 8 & 3 & 2 \\
7 & 8 & 9 & 3 & 5 & 2 & 1 & 4 & 6 \\
\end{array}
\]

2. Distinguish cases for first row:

(Type I)

\[
\begin{array}{ccc|ccc|ccc}
1 & 2 & 3 & 4,5,6 & 7,8,9 \\
4 & 5 & 6 & 7,8,9 & 1,2,3 \\
7 & 8 & 9 & 1,2,3 & 4,5,6 \\
\end{array}
\]
Symmetry Breaking

We “break the symmetry” by considering exactly one from each equivalence class. Consider only first block row:

1 We can renumber to get this left hand 3 × 3-block:

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<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>6</th>
<th>8</th>
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<td>2</td>
<td>1</td>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

2 Distinguish cases for first row:

(Type I)

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4,5,6</th>
<th>7,8,9</th>
</tr>
</thead>
<tbody>
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<td>4</td>
<td>5</td>
<td>6</td>
<td>7,8,9</td>
<td>1,2,3</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
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<td>1,2,3</td>
<td>4,5,6</td>
</tr>
</tbody>
</table>

(Type II)

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4,5,7</th>
<th>6,8,9</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>5</td>
<td>6</td>
<td>4,5,7</td>
<td></td>
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<tr>
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<td>9</td>
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</tbody>
</table>
Symmetry Breaking

We “break the symmetry” by considering exactly one from each equivalence class. Consider only first block row:

1. We can renumber to get this left hand $3 \times 3$-block:

2. Distinguish cases for first row:

(Type I)

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>{4,5,6}</th>
<th>{7,8,9}</th>
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<tr>
<td>4</td>
<td>5</td>
<td>6</td>
<td>{7,8,9}</td>
<td>{1,2,3}</td>
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<tr>
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<td>8</td>
<td>9</td>
<td>{1,2,3}</td>
<td>{4,5,6}</td>
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</tbody>
</table>

(Type II)

<table>
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<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>{4,5,7}</th>
<th>{6,8,9}</th>
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<tr>
<td>4</td>
<td>5</td>
<td>6</td>
<td>{8,9,a}</td>
<td>{7,b,c}</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
<td>9</td>
<td>{6,b,c}</td>
<td>{4,5,a}</td>
</tr>
</tbody>
</table>

where \( \{a, b, c\} = \{1, 2, 3\} \).
Symmetry Breaking

We “break the symmetry” by considering exactly one from each equivalence class. Consider only first block row:

1. We can renumber to get this left hand $3 \times 3$-block:

```
1 2 3  |  4 6 8  |  9 5 7  
4 5 6  |  9 1 7  |  8 3 2  
7 8 9  |  3 5 2  |  1 4 6  
```

2. Distinguish cases for first row:

- **(Type I)**

```
1 2 3  | \{4,5,6\} | \{7,8,9\} 
4 5 6  | \{7,8,9\} | \{1,2,3\} 
7 8 9  | \{1,2,3\} | \{4,5,6\} 
```

- **(Type II)**

```
1 2 3  | \{4,5,7\} | \{6,8,9\} 
4 5 6  | \{8,9,a\} | \{7,b,c\} 
7 8 9  | \{6,b,c\} | \{4,5,a\} 
```

where $\{a, b, c\} = \{1, 2, 3\}$.

3. Some more such arguments ...
Is there a Sudoku puzzle with 16 clues?

Max Neunhöffer

The Problem

What is a Sudoku?
How many clues?

Symmetry
Equivalent Sudokus
Symmetry Breaking

Backtrack Search
Unavoidable Sets
The Hitting Set Problem
Backtrack Search

Random Search
Finding Unavoidable Sets

The Result

Unavoidable Sets

Question

Fix one Sudoku grid. Can it be the solution to a 16-clue Sudoku puzzle?
Unavoidable Sets

Question
Fix one Sudoku grid. Can it be the solution to a 16-clue Sudoku puzzle?

Number of ways to choose 16 out of 81:

\[
\binom{81}{16} = 33\,594\,090\,947\,249\,085 \approx 33 \cdot 10^{15}
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Unavoidable Sets

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⇒ do not even think about trying all!
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\[\implies \text{do not even think about trying all!}\]

Idea: We do not have to try all choices.
Unavoidable Sets

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Fix one Sudoku grid. Can it be the solution to a 16-clue Sudoku puzzle?

Number of ways to choose 16 out of 81:

$$\binom{81}{16} = 33\,594\,090\,947\,249\,085 \approx 33 \cdot 10^{15}$$

⇒ do not even think about trying all!

Idea: We do not have to try all choices.

We need constraints that the selection of 16 has to fulfil.
Unavoidable Sets

Definition: Unavoidable Set

Let $S$ be a filled Sudoku grid. A subset $U$ of the 81 positions is called an **unavoidable set**, if every set of clues uniquely defining $S$ has a number in at least one of the positions in $U$. 
Unavoidable Sets

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Questions:

- Are there unavoidable sets and if so how many?
Unavoidable Sets

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- How can we find them?
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Questions:

- Are there unavoidable sets and if so how many?
- How can we find them?
- How does this help?
Is there a Sudoku puzzle with 16 clues?

Max Neunhöffer

The Problem

What is a Sudoku?

How many clues?

Symmetry

Equivalent Sudokus

Symmetry Breaking

Backtrack Search

Unavoidable Sets

The Hitting Set Problem

Backtrack Search

Random Search

Finding Unavoidable Sets

The Result

Unavoidable sets

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Any set of 16 clues cannot avoid the yellow positions.

Because this puzzle has more than one solution.
Unavoidable sets

Any set of 16 clues cannot avoid the yellow positions.
Is there a Sudoku puzzle with 16 clues?

Max Neunhöffer

The Problem
What is a Sudoku?
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Symmetry
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Symmetry Breaking

Backtrack Search
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The Hitting Set Problem
Backtrack Search

Random Search
Finding Unavoidable Sets

The Result

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Any set of 16 clues cannot avoid the yellow positions. Because this puzzle has more than one solution.
The Hitting Set Problem

Problem: Hitting Set (resp. Set Covering)

Let $M$ be a set and let $A_1, \ldots, A_k$ be subsets of $M$. Find a minimal subset $H$ of $M$ which contains at least one element of every $A_i$ for $1 \leq i \leq k$. 

This problem is computationally hard. It is one of Karp's 21 NP-complete problems ([1]). We want to solve it to use lots of unavoidable sets to reduce the number of 16-clue sets we need to consider.

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This problem is \textbf{computationally hard}.

It is one of Karp’s 21 \textbf{NP-complete} problems (→ [1]).

We want to solve it to use lots of \textbf{unavoidable sets} to \textbf{reduce} the number of 16-clue sets we need to consider.

Algorithm: Hitting Sets

An example Hitting Set Problem

Let \( M = \{1, 2, 3, \ldots, 12\} \) and
\[
\mathcal{A} := \{ \{1, 2, 6\}, \\
\{2, 3, 4, 8\}, \\
\{1, 7, 8, 9\}, \\
\{3, 4, 6, 9\}, \\
\{4, 6, 10, 12\}, \\
\{2, 10, 11, 12\}, \\
\{5, 7, 8, 9\}, \\
\{5, 7, 10, 12\}, \\
\{1, 3, 4, 5, 11\} \}
\]

Find a 3-subset of \( M \) intersecting all members of \( \mathcal{A} \) non-trivially.
Is there a Sudoku puzzle with 16 clues?

Max Neunhöffer

The Problem
What is a Sudoku?
How many clues?

Symmetry
Equivalent Sudokus
Symmetry Breaking

Backtrack Search
Unavoidable Sets
The Hitting Set Problem

Random Search
Finding Unavoidable Sets

The Result

Backtrack Search in Action

Start

1 2 6

1st number chosen
Is there a Sudoku puzzle with 16 clues?

Max Neunhöffer

The Problem
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How many clues?

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The Result

Backtrack Search in Action

Start

1

2

6

1st number chosen

2nd number chosen

2 3 4 8

2nd number chosen

1st number chosen

1 6 2

1st number chosen

2nd number chosen
Is there a Sudoku puzzle with 16 clues?

Max Neunhöffer

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Random Search
Finding Unavoidable Sets

The Result

Backtrack Search in Action

Start

1st number chosen

2nd number chosen

3rd number chosen

Random Search
Finding Unavoidable Sets

The Result
Backtrack Search in Action

Start

1st number chosen
2nd number chosen
3nd number chosen

all no good
Is there a Sudoku puzzle with 16 clues?

Max Neunhöffer

The Problem
What is a Sudoku?
How many clues?

Symmetry
Equivalent Sudokus
Symmetry Breaking

Backtrack Search
Unavoidable Sets
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Backtrack Search

Random Search
Finding Unavoidable Sets

The Result

Backtrack Search in Action

Start

1st number chosen

2nd number chosen

3rd number chosen

1 6 2
2 3 4 8
3 4 6 9
10 11 12

all no good
2 and 3 not needed

121110
not needed
2 and 3
Is there a Sudoku puzzle with 16 clues?
Max Neunhöffer

Backtrack Search in Action

Start

1
2 no 1
6 no 1,2

2
3
4
8
7
8
9

... 2nd

3
4
6
9
10
11
12
3
4
6
9

all no good
2 and 3 not needed

Unique solution: \{2,4,7\}

Hurrah!

2nd
Proof that it works

Problem: Hitting Set (resp. Set Covering)

Let \( M \) be a set and let \( A_1, \ldots, A_k \) be subsets of \( M \). Find a minimal subset \( H \) of \( M \) which contains at least one element of every \( A_i \) for \( 1 \leq i \leq k \).

We need to prove that every solution \( H \) is found traversing the tree!
Is there a Sudoku puzzle with 16 clues?

Max Neunhöffer

The Problem

What is a Sudoku?
How many clues?

Symmetry

Equivalent Sudoku
Symmetry Breaking

Backtrack Search

Unavoidable Sets
The Hitting Set Problem

Random Search

Finding Unavoidable Sets

The Result

Proof that it works

Problem: Hitting Set (resp. Set Covering)

Let $M$ be a set and let $A_1, \ldots, A_k$ be subsets of $M$.
Find a minimal subset $H$ of $M$ which contains at least one element of every $A_i$ for $1 \leq i \leq k$.

We need to prove that every solution $H$ is found traversing the tree!

Proof: It works!

Let $H$ be a solution. Then it intersects all $A_i$ for $1 \leq i \leq k$.
It is found in exactly one leaf of the tree!
Algorithm: Finding Unavoidable Sets

Definition: Unavoidable Set

Let $S$ be a filled Sudoku grid. A subset $U$ of the 81 positions is called an **unavoidable set**, if every set of clues uniquely defining $S$ has a number in at least one of the positions in $U$. 
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That is: If we leave out the numbers in the positions $U$, there is more than one solution.
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Algorithm: Start with a full grid.

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Algorithm: Start with a full grid.
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3. Try to put back each number to ensure minimality.

Improvement: Whenever our candidate set $C$ contains an already known unavoidable set $U$, we remove $U$ from $C$. 
Performance for our problem

I have run this method on all 49151 solutions of the 17-clue Sudoku puzzles collected by Gordon Royle.
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The good news:

- Usually finds 2048 unavoidable sets in $\approx 10$ s.
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- The **Hitting Set Problem** has **very few solutions**.
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- If I go on like this with the 5 472 730 538,
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The bad news:

If I go on like this with the 5 472 730 538,

I need another 300 000 CPU years.